Exam in *Model Checking*

February 10, 2006

Family name: ______________________________

First name: ______________________________

Student number: ______________________________

Please note the following hints:

- Keep your student id card and a passport ready.
- The only allowed materials are
  - a copy of the lecture notes and
  - a copy of the lecture slides.

  No other materials (i.a. exercises, solutions, handwritten notes) are admitted.
- This test should have six pages (including this cover sheet).
- Write your name and student number on every sheet.
- Also use the back side of the pages if needed.
- Write with blue or black ink; do not use a pencil.
- Any attempt at deception leads to failure for this exam, even if it is detected only later.
- The editing time is **90 minutes**.

Question	Possible	Received
1	10	
2	10	
3	10	
4	10	
5	10	
Total	50	
Grade		

Question 1

(10 points)

Let P be a linear time property. Prove that $closure(P)$ is a safety property.

Question 2 (10 points)

Let $\varphi = (a \rightarrow \bigcirc\neg b)\mathsf{W}(a \wedge b)$ and $P = Words(\varphi)$ where $AP = \{a, b\}$.

(a) Show that P is a safety property.

(b) Define an NFA $\mathcal{A}$ with $\mathcal{L}(\mathcal{A}) = BadPref(P)$.

(c) Now consider $P' = Words\big((a \rightarrow \bigcirc\neg b)\mathsf{U}(a \wedge b)\big)$.
Decompose P' into a safety property P_{safe} and a liveness property P_{live} such that

$$P' = P_{safe} \cap P_{live}.$$

Show that P_{safe} is a safety and that P_{live} is a liveness property.

Question 3 (10 points)

Let $\psi = \Box\,(a \leftrightarrow \bigcirc\neg a)$ and $AP = \{a\}$.

(a) Show that ψ can be transformed into the following equivalent basic LTL-formula

$$\varphi = \neg\left[\mathsf{true}\,\mathsf{U}\big(\neg\,(a \wedge \bigcirc\neg a) \wedge \neg\,(\neg a \wedge \neg \bigcirc \neg a)\,\big)\right].$$

The basic LTL syntax is given by the following context free grammar:

$$\varphi ::= \mathsf{true} \mid a \mid \varphi_1 \wedge \varphi_2 \mid \neg\varphi \mid \bigcirc\varphi \mid \varphi_1 \mathsf{U} \varphi_2$$

(b) Compute all elementary sets with respect to $closure(\varphi)$!
Hint: There are 6 elementary sets.

(c) Use the algorithm from the lecture to construct the GNBA $\mathcal{G}_\varphi$ with $\mathcal{L}_\omega(\mathcal{G}_\varphi) = Words(\varphi)$. Therefore

- define its set of initial states and its acceptance component.
- for each elementary set B, define $\delta(B, B \cap AP)$!

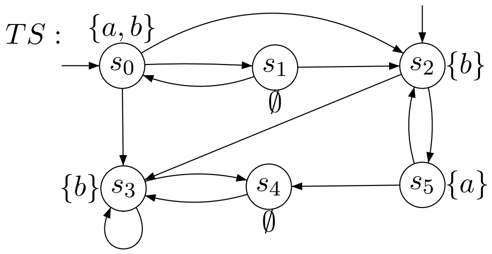
Question 4

(10 points)

Compute $Sat_{sfair}(\Phi)$ for the CTL-formula Φ and the strong fairness assumption $sfair$:

$$\Phi = \forall\Box\forall\Diamond a$$
$$sfair = \Box\Diamond\underbrace{(b \wedge \neg a)}_{\Phi_1} \rightarrow \Box\Diamond\underbrace{\exists(b\,\mathsf{U}(a \wedge \neg b))}_{\Psi_1}$$

where TS over $AP = \{a, b\}$ is given by:



Therefore

(a) Determine $Sat(\Phi_1)$ and $Sat(\Psi_1)$ (without fairness).

(b) Determine $Sat_{sfair}(\exists\Box\,\mathsf{true})$.

(c) Determine $Sat_{sfair}(\Phi)$.

Question 5

(10 points)

For all $0 < i < j \leq 3$, prove or disprove $TS_i \sim TS_j$ either by providing a bisimulation relation or by providing a distinguishing CTL-formula Φ (i.e. $TS_i \models \Phi \iff TS_j \not\models \Phi$), respectively: