Winter term 2008/09

Prof. Dr. Ir. J.-P. Katoen

Exam in *Model Checking* for Master of Science and diploma students

March 30, 2009

Family name: ____________________

First name: ____________________

Student number: ____________________

Field of study:
- ☐ **Informatik (Diplom)**
- ☐ **Software Systems Engineering**
- ☐ **Others:** ____________________

Please note the following hints:

- Keep your student id card and a passport ready.
- The only allowed materials are
  - a copy of the lecture notes,
  - a copy of the lecture slides and
  - a dictionary.

  No other materials (i.a. exercises, solutions, handwritten notes) are admitted.
- This test should have six pages (including this cover sheet).
- Write your name and student number on every sheet.
- Also use the back side of the pages if needed.
- Write with blue or black ink; do not use a pencil.
- Any attempt at deception leads to failure for this exam, even if it is detected only later.
- The editing time is **90 minutes**.

Question	Possible	Received
1	10	
2	10	
3	10	
4	10	
5	10	
Total	50	
Grade		

Question 1

(10 points)

Let P be a linear time property. Prove that P is a liveness property *if and only if* $closure(P) = \left(2^{AP}\right)^{\omega}$.

Question 2 (10 points)

Let P denote the linear time property over the set $AP = \{a, b\}$ of atomic propositions such that P consists of all infinite traces $\sigma = A_0 A_1 A_2 \cdots \in \left(2^{AP}\right)^\omega$ that satisfy

$$\forall i \geq 0.\ (A_i = \emptyset \implies \exists k \geq i.\ (b \in A_k \wedge \forall j \in \{i, \ldots, k-1\}\ .\ a \notin A_j))\,.$$

(a) Specify an LTL formula φ such that $\mathit{Words}(\varphi) = P$.

(b) Give an ω–regular expression for P.

(c) Apply the decomposition theorem and give ω–regular expressions for P_{safe} and P_{live}.

Question 3 (10 points)

Let $\varphi = (a \wedge \bigcirc a)\mathsf{U}(\neg(\neg a \mathsf{U} a))$ be a LTL formula over $AP = \{a\}$.

(a) Compute all elementary sets with respect to *closure*(φ)!
Hint: There are 7 elementary sets.

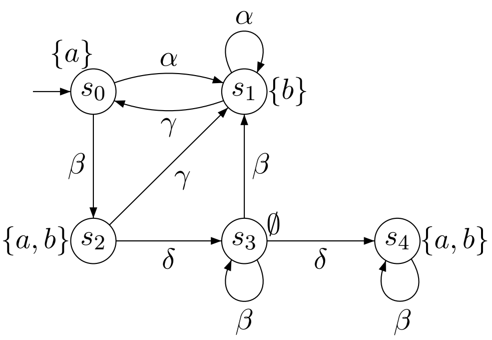
(b) Use the algorithm from the lecture to construct the GNBA $\mathcal{G}_\varphi$ with $\mathcal{L}_\omega(\mathcal{G}_\varphi) = \mathit{Words}(\varphi)$:

- Define the set of initial states and the acceptance component.
- Depict the transition relation of $\mathcal{G}_\varphi$.
  Hint: It suffices to consider the reachable elementary sets only!

(c) Informally describe the language $\mathcal{L}_\omega(\mathcal{G}_\varphi)$.

Question 4 (10 points)

Let P denote the set of traces $\sigma = A_0 A_1 A_2 \cdots \in \left(2^{AP}\right)^\omega$ over $AP = \{a, b\}$ such that there exist infinitely many indices $k \geq 0$ with $A_k = \emptyset$. Consider the following transition system TS:



For each of the fairness assumptions

(a) $\mathcal{F}_1 = \big(\{\{\alpha\}\}, \{\{\beta\}, \{\delta, \gamma\}\}, \emptyset\big)$ and

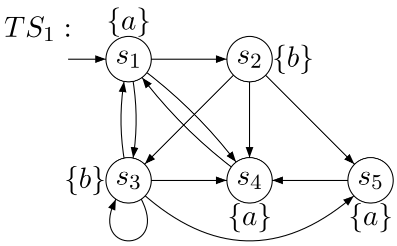
(b) $\mathcal{F}_2 = \big(\{\{\alpha\}\}, \{\{\beta\}, \{\delta\}, \{\gamma\}\}, \emptyset\}\big)$:

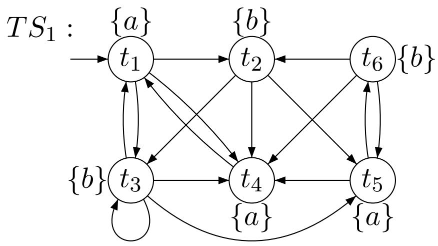
Decide whether $TS \models_{\mathcal{F}_i} P$ for $i = 1, 2$. Justify your answers!

Question 5

(10 points)

Consider the following transition systems TS_1 and TS_2:





(a) Compute $TS_1/\sim$ and $TS_2/\sim$.

(b) Decide whether $TS_1 \sim TS_2$. Explain your answer.